Practica 8

Puntos Críticos
y
Clasificación.

Problema 1

Para las siguientes funciones encuentre sus puntos críticos locales y clasifíquelos.

a) $f(x,y) = x^2 + xy + y^2 - 3y + 4$

b) $f(x,y) = x^2 + 3xy + 3y^2 - 6x + 3y - 6$

c) $f(x,y) = \dfrac{1}{x^2 + y^2 - 1}$

Ejercicio 1

a) $f(x, y) = x^2 + xy + y^2 - 3y + 4$

$\nabla f(x, y) = 0$ → $\begin{cases} f_x(x, y) = 2x + y + 3 = 0 \\ f_y(x, y) = x + 2y - 3 = 0 \end{cases}$ → $(-3, 3)$

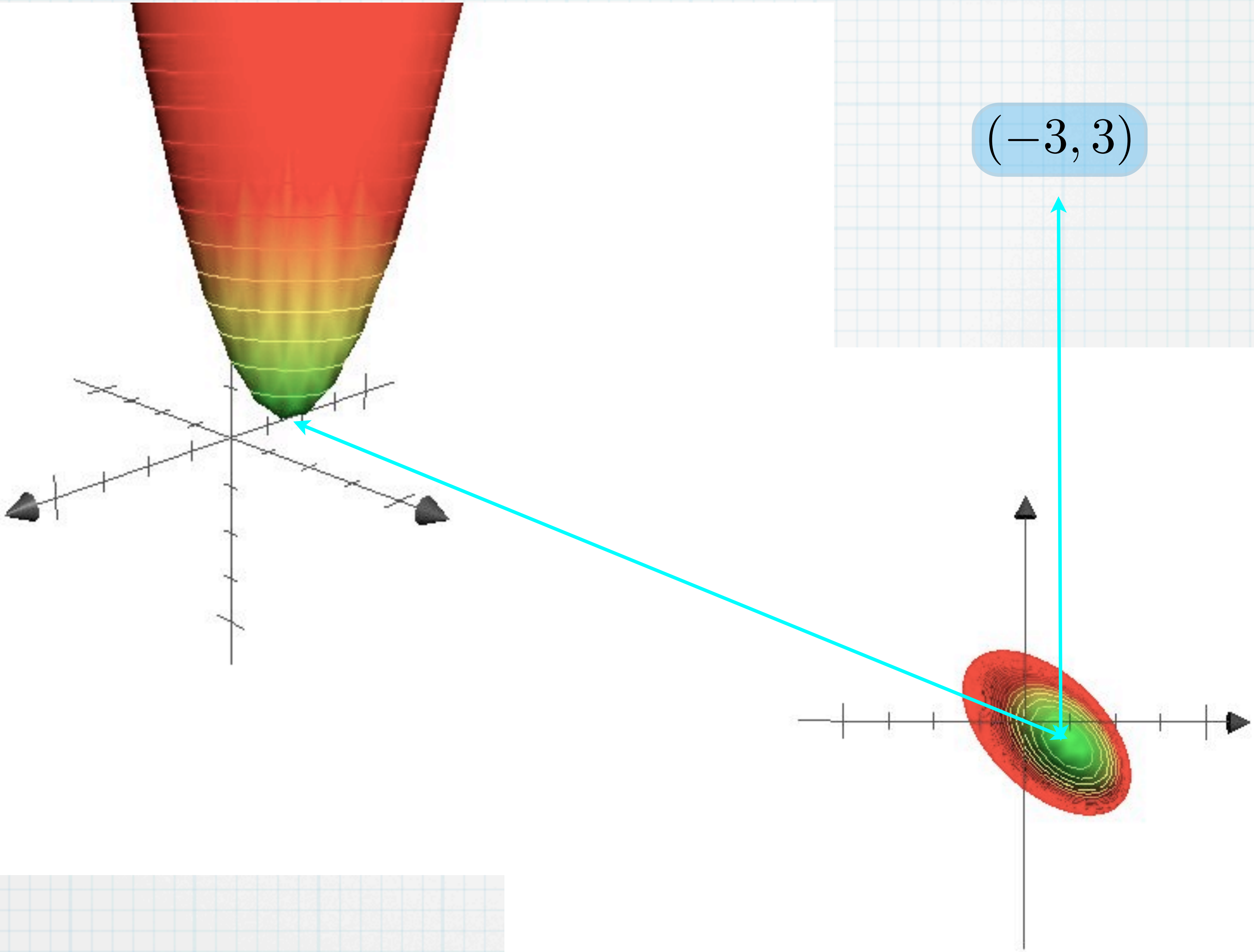
$f_{xx}(-3, 3) = 2$

$f_{yy}(-3, 3) = 2$

$f_{xy}(-3, 3) = 1$

→ $f_{xx}f_{yy} - f_{xy}^2 = 3 > 0$, $f_{xx} > 0$ → **Mínimo local**

$f(-3, 3) = -5$ **Valor del mínimo**


$(-3, 3)$

b) $$f(x,y) = x^2 + 3xy + 3y^2 - 6x + 3y - 6$$

$$\nabla f(x,y) = 0 \;\Rightarrow\; \begin{cases} f_x(x,y) = 2x + 3y - 6 = 0 \\ f_y(x,y) = 3x + 6y + 3 = 0 \end{cases} \;\Rightarrow\; (15, -8)$$

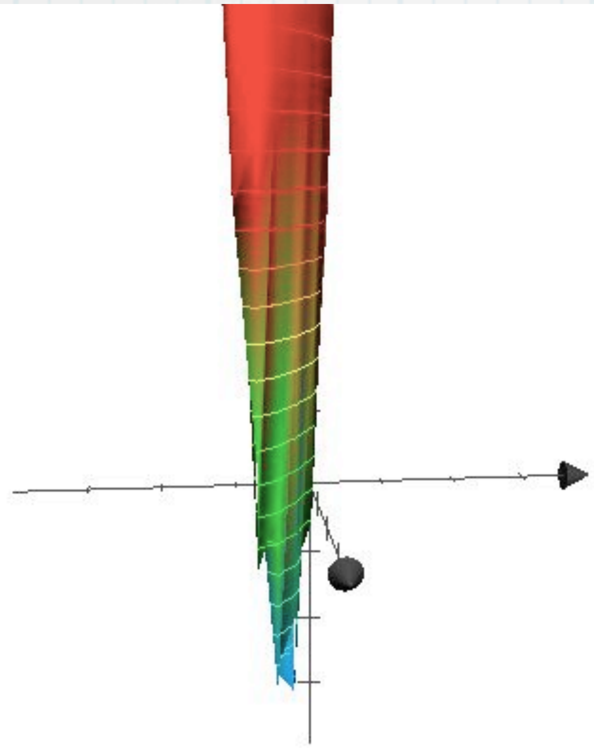
$f_{xx}(15,-8) = 2,$

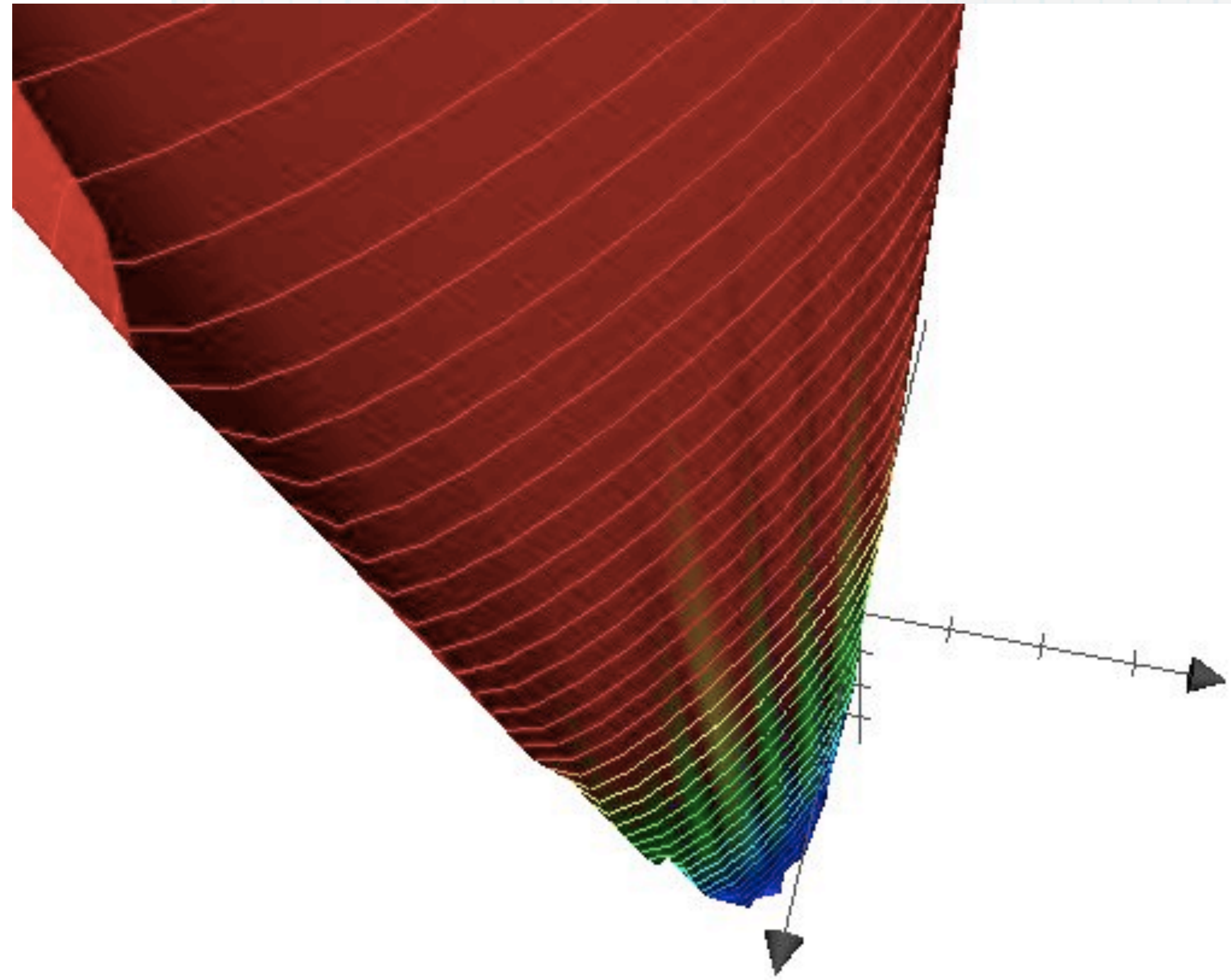
$f_{yy}(15,-8) = 6,$

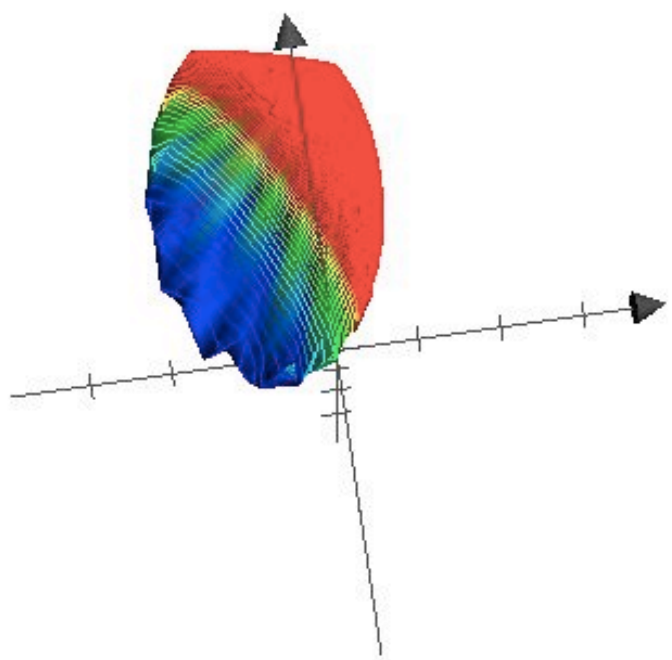
$f_{xy}(15,-8) = 3$

$$\Rightarrow\; \begin{cases} f_{xx}f_{yy} - f_{xy}^2 = 3 > 0 \\ f_{xx} > 0 \end{cases} \;\Rightarrow\; \textbf{Mínimo local}$$

$f(15,-8) = -63$ **Valor del mínimo**







c) $$f(x,y) = \frac{1}{x^2+y^2-1}$$

$$\nabla f(x,y) = 0$$ →

$$f_x(x,y) = \frac{-2x}{(x^2+y^2-1)^2} = 0$$

$$f_y(x,y) = \frac{-2y}{(x^2+y^2-1)^2} = 0$$

→ $(0,0)$ **Punto crítico**

$$f_{xx} = \frac{4x^2-2y^2+2}{(x^2+y^2-1)^3},$$

$$f_{yy} = \frac{-2x^2+4y^2+2}{(x^2+y^2-1)^3},$$

$$f_{xy} = \frac{8xy}{(x^2+y^2-1)^3}$$

→

$$f_{xx}(0,0) = -2,$$
$$f_{yy}(0,0) = -2,$$
$$f_{xy}(0,0) = 0$$

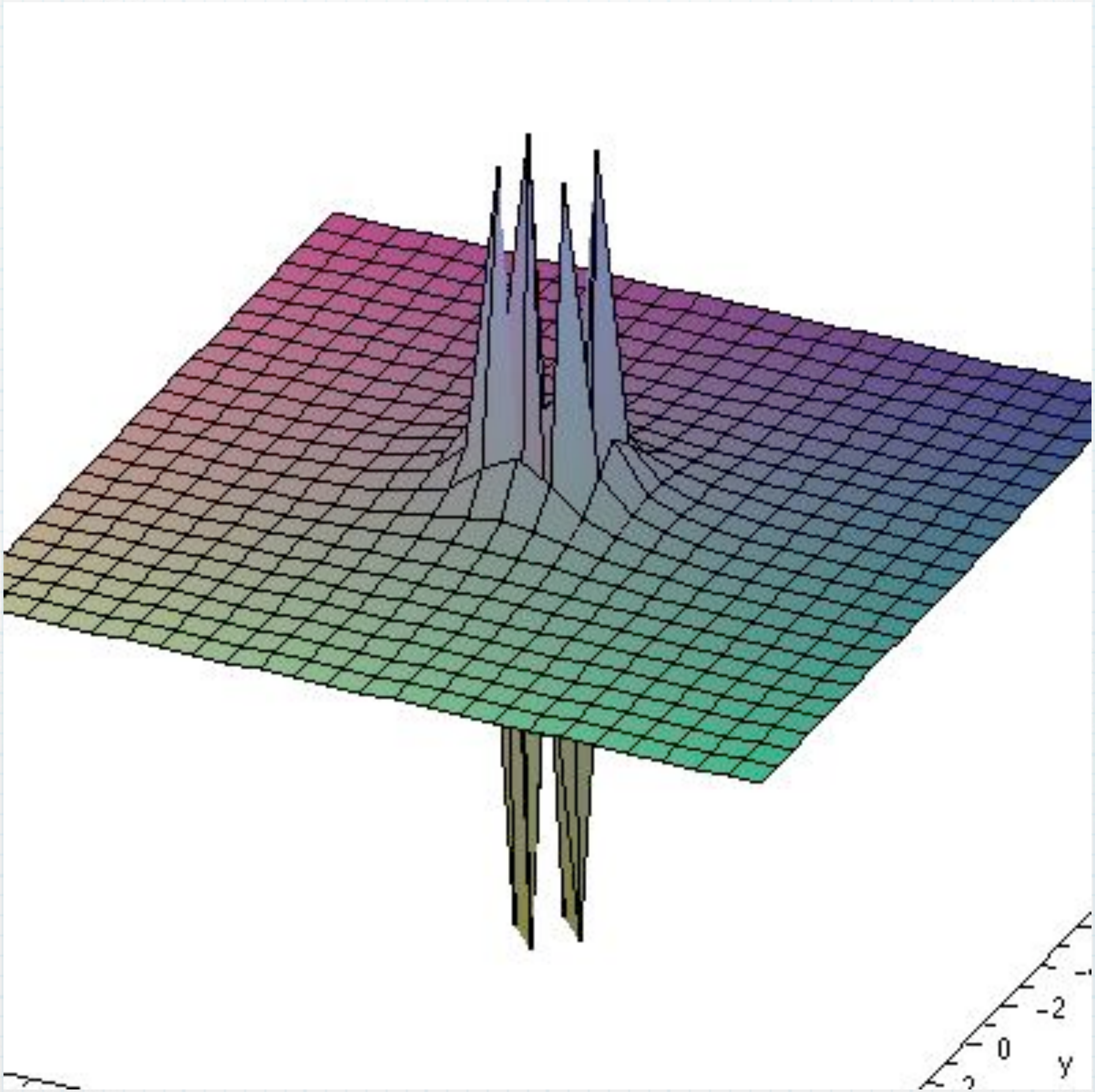
→

$$f_{xx}f_{yy} - f_{xy}^2 = 4 > 0$$

$$f_{xx} < 0$$

→ **Máximo local**

$f(0,0) = -1$ **Valor del máximo**


-2
0
y

problema 2.

Encuentre los extremos absolutos y clasifíquelos, para las siguientes funciones

a) $f(x,y) = 2x^2 + 4x + y^2 - 4y + 1$

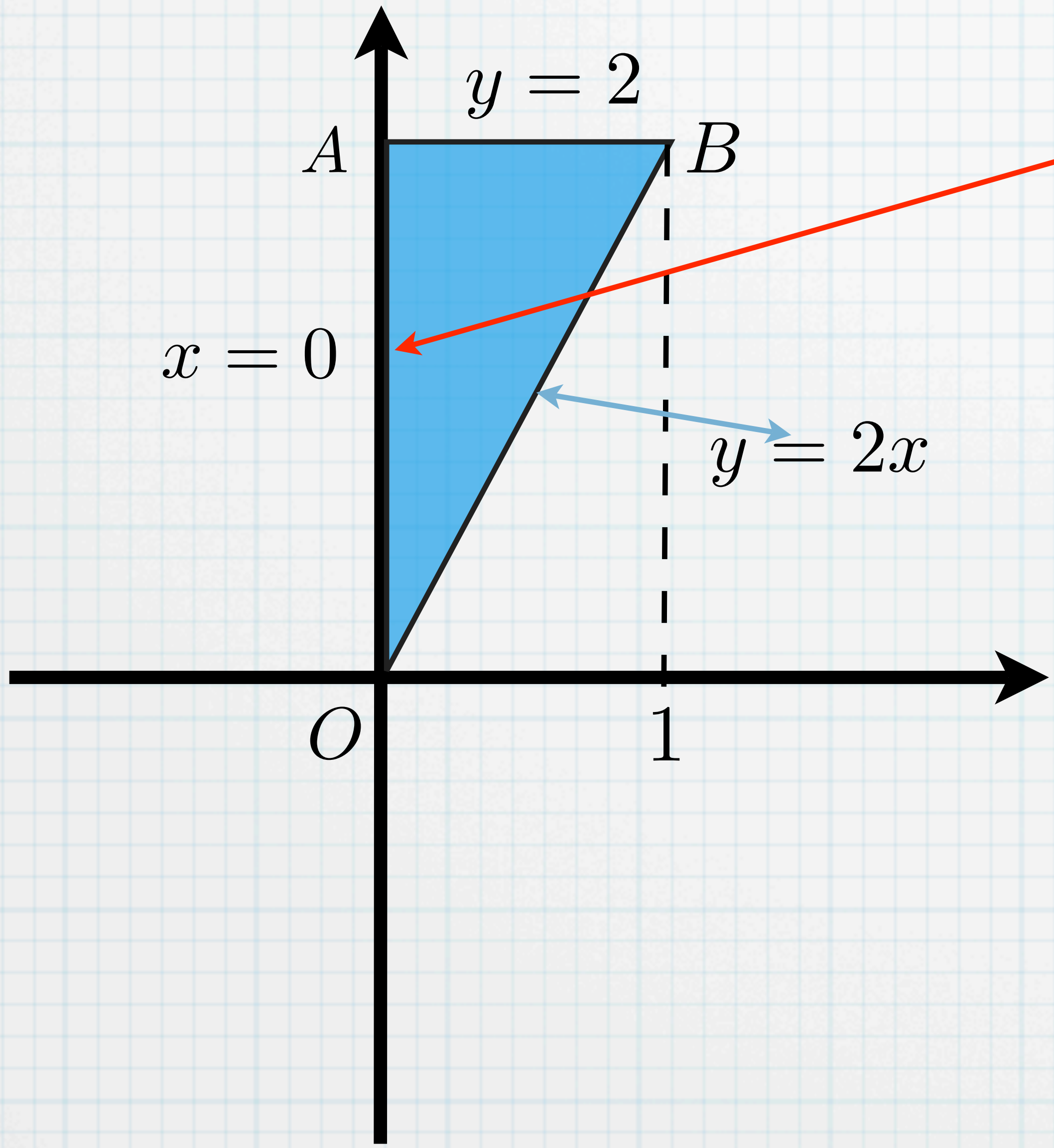
En la región delimitada por $x = 0, \; y = 2, \; y = 2x$, que se encuentra en el primer cuadrante

b) $f(x,y) = (4x - x^2)\cos(y)$

En la región delimitada por $1 \leq x \leq 3, \; \frac{-\pi}{4} \leq y \leq \frac{\pi}{4}$

a) $f(x,y) = 2x^2 + 4x + y^2 - 4y + 1$

$x = 0, \;\; y = 2, \;\; y = 2x \,,$



i) En OA:

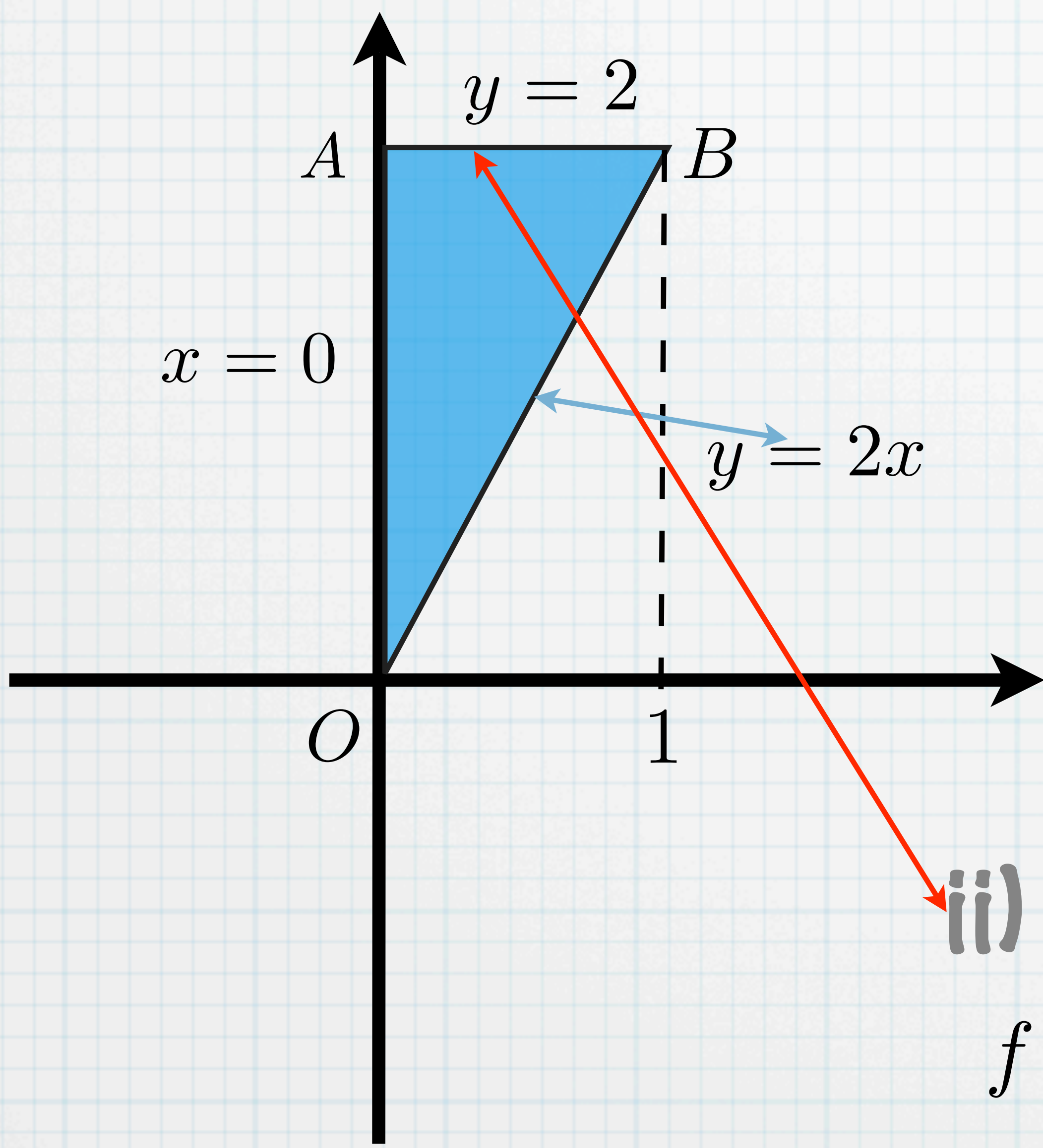
$f(o,y) = y^2 - 4y + 1 \text{ en } 0 \leq y \leq 2$

$f'(o,y) = 2y - 4 = 0 \Rightarrow y = 2$

$f(0,0) = 1 \text{ y } f(0,2) = -3$

a) $f(x,y) = 2x^2 + 4x + y^2 - 4y + 1$

$x = 0, \;\; y = 2, \;\; y = 2x \,,$



i) En OA:

$f(o,y) = y^2 - 4y + 1$ en $0 \leq y \leq 2$

$f'(o,y) = 2y - 4 = 0 \Rightarrow y = 2$

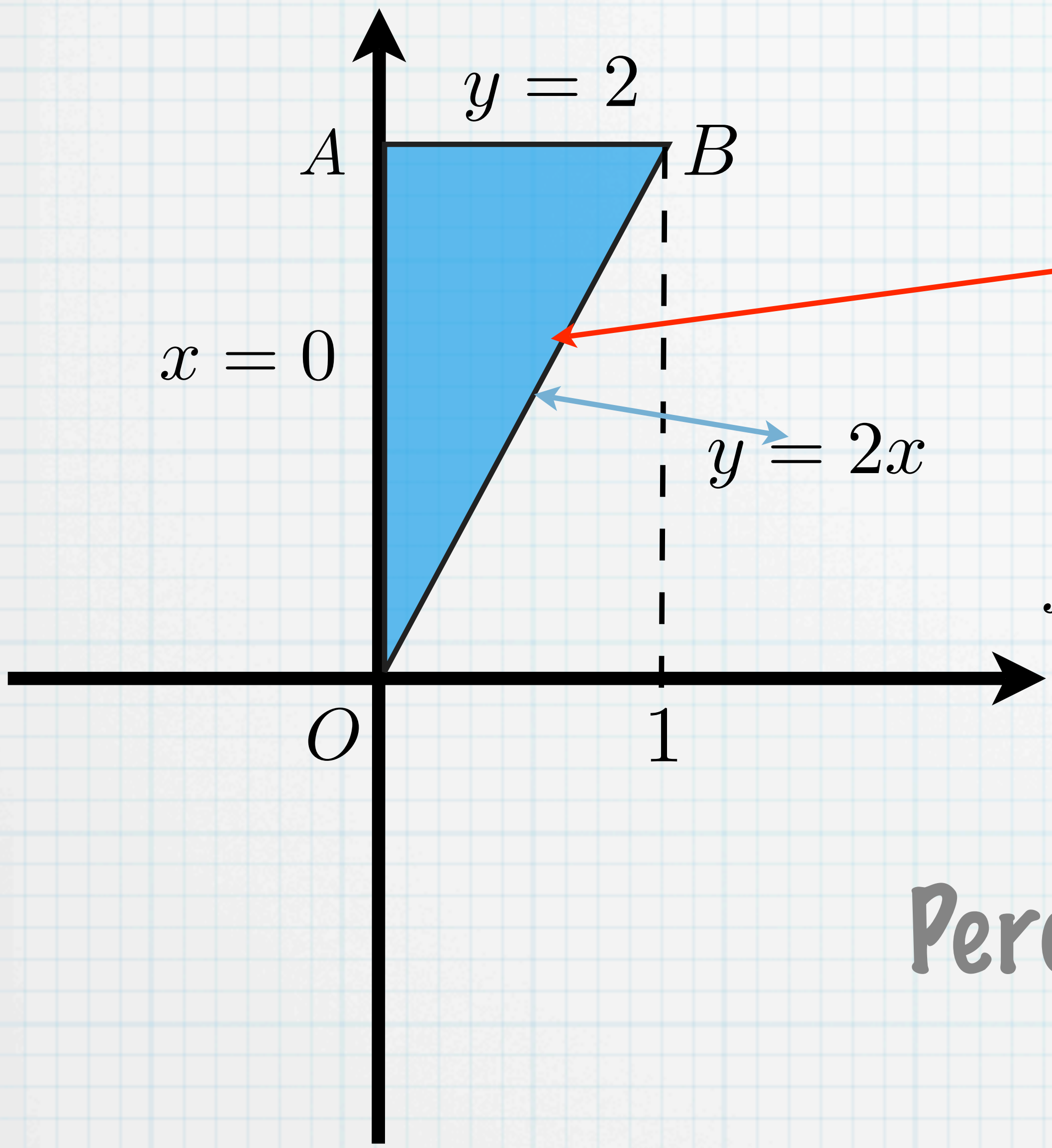
$f(0,0) = 1$ y $f(0,2) = -3$

ii) En AB:

$f(x,2) = 2x^2 - 4x - 3$

$f'(x,2) = 4x - 4 = 0 \Rightarrow x = 1$

$f(0,2) = -3$ y $f(1,2) = -5$

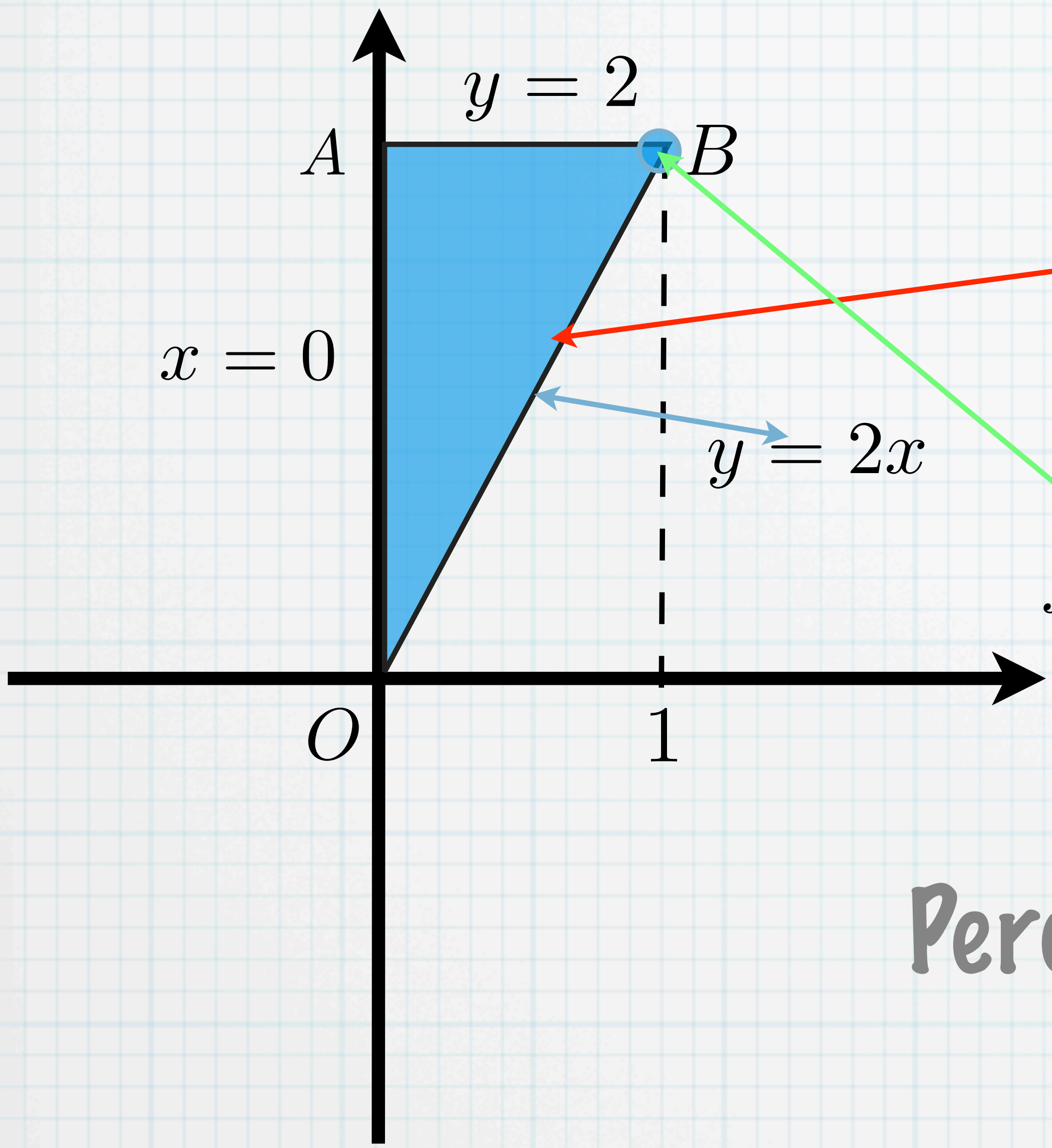


iii) En OB:

$$f(x, 2x) = 6x^2 - 12x + 1, \text{ en } 0 \leq x \leq 1$$

$$f'(x, 2x) = 12x - 12 = 0 \Rightarrow x = 1 \text{ y } y = 2$$

Pero el punto (1,2) esta fuera de y=2x

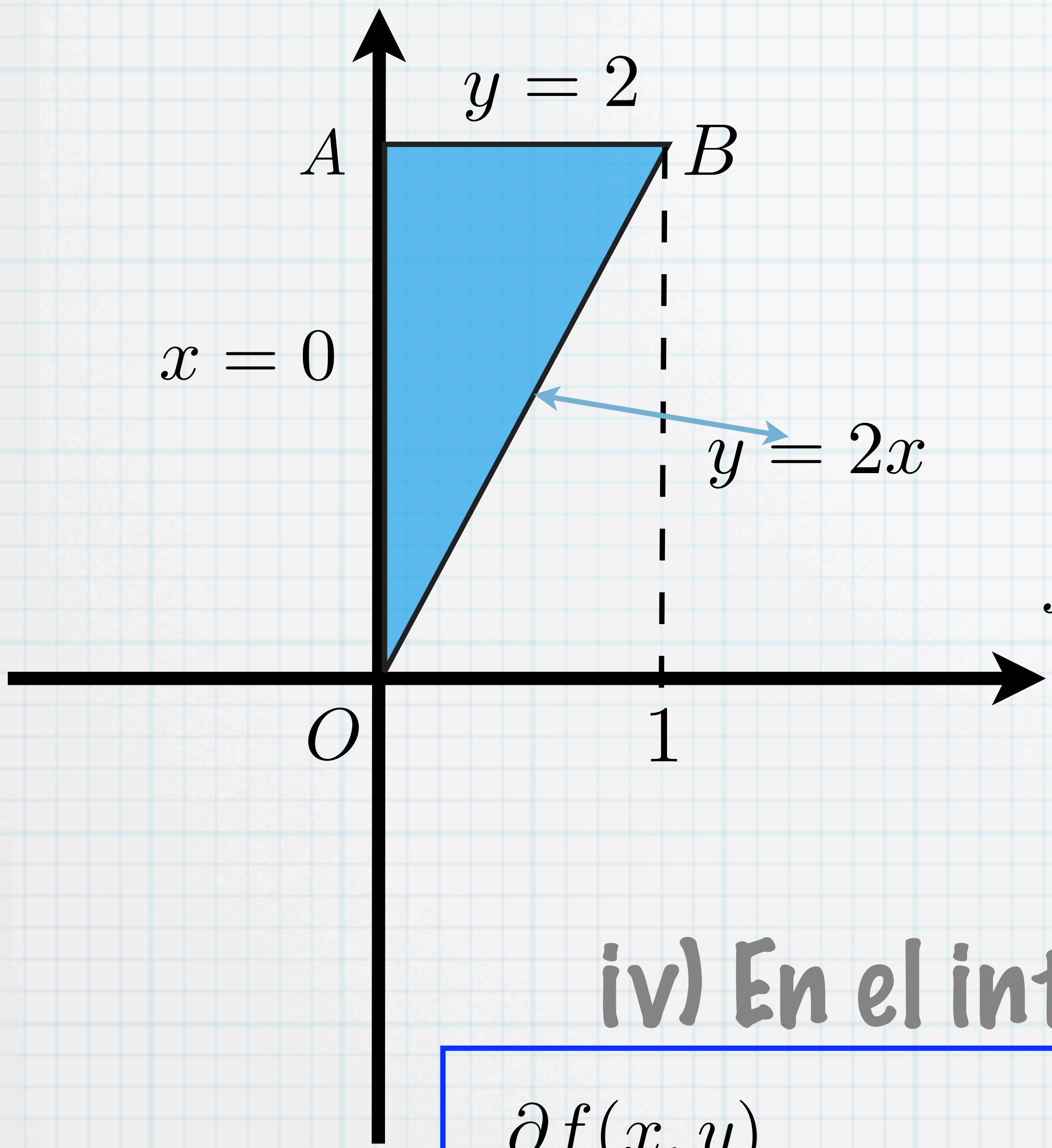


iii) En OB:

$$f(x, 2x) = 6x^2 - 12x + 1, \text{ en } 0 \leq x \leq 1$$

$$f'(x, 2x) = 12x - 12 = 0 \Rightarrow x = 1 \text{ y } y = 2$$

Pero el punto (1,2) esta fuera de y=2x



iii) En OB:

$$f(x, 2x) = 6x^2 - 12x + 1, \text{ en } 0 \leq x \leq 1$$

$$f'(x, 2x) = 12x - 12 = 0 \Rightarrow x = 1 \text{ y } y = 2$$

Pero el punto (1,2) esta fuera de y=2x

iv) En el interior:

$$\frac{\partial f(x,y)}{\partial x} = 4x - 4 = 0$$

$$\frac{\partial f(x,y)}{\partial y} = 2y - 4 = 0$$

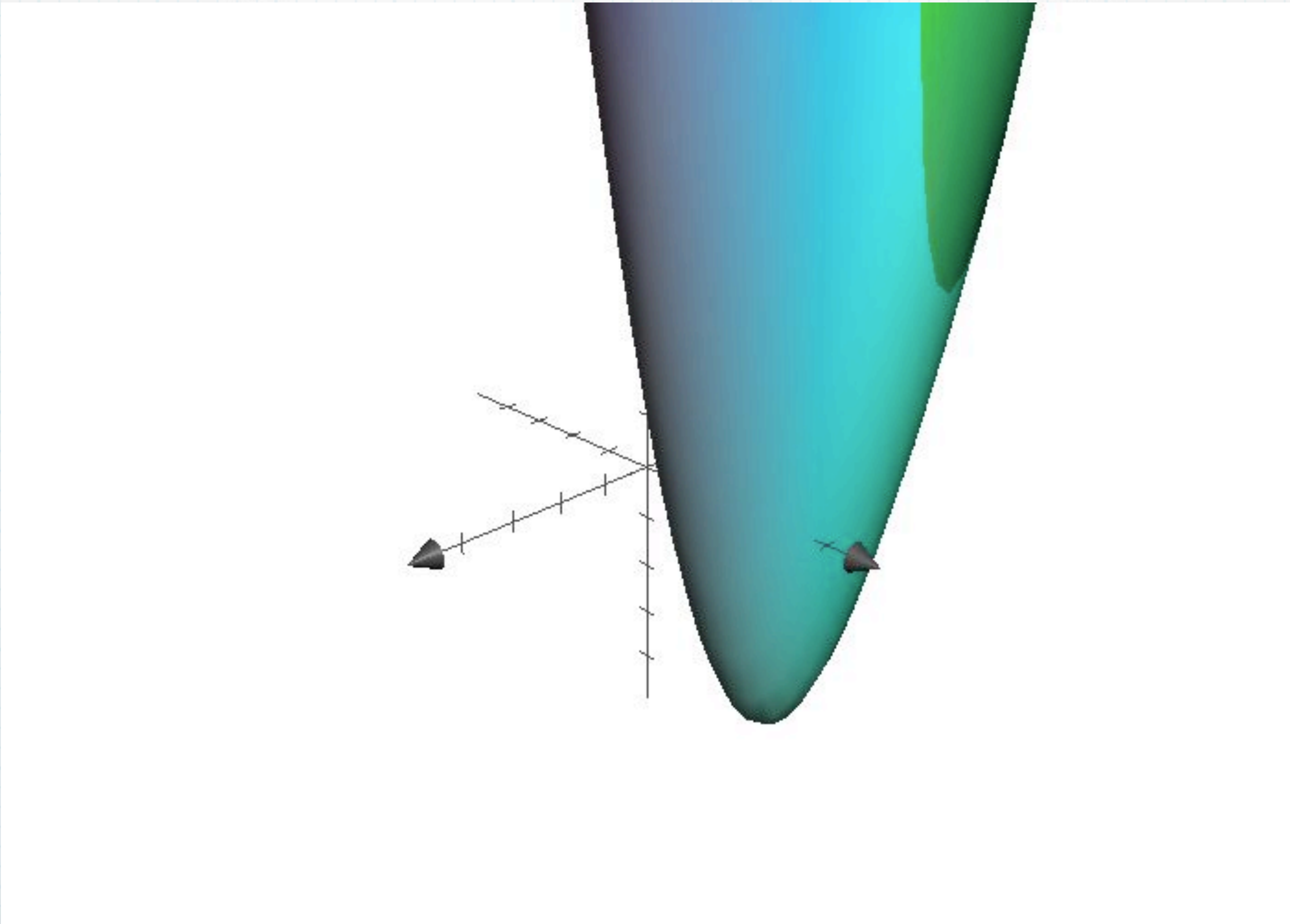
$\Rightarrow$ $x = 1, \text{ y } y = 2$

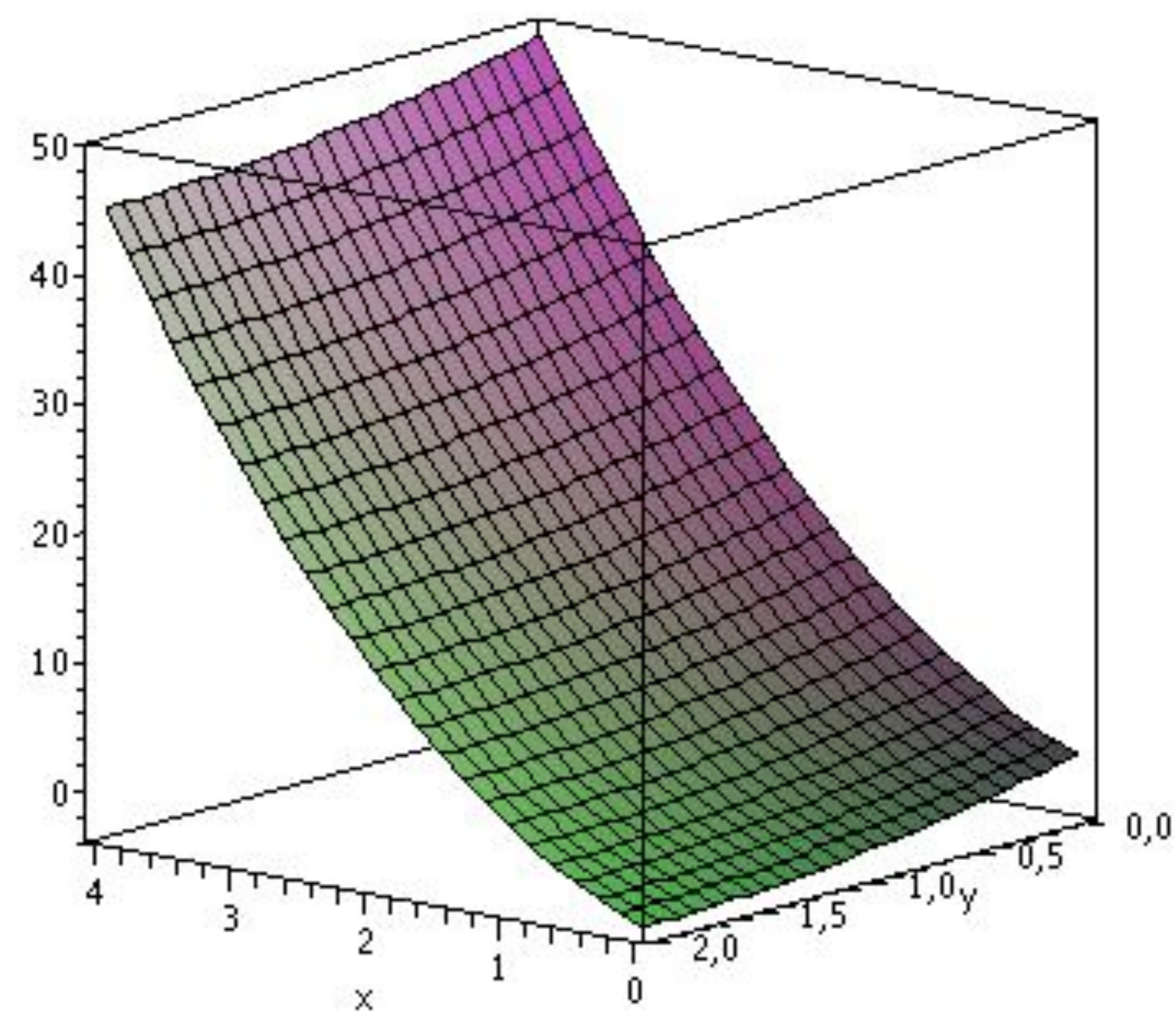
$$(1, 2)$$

Finalmente:

1 es el máximo absoluto en (0,0)

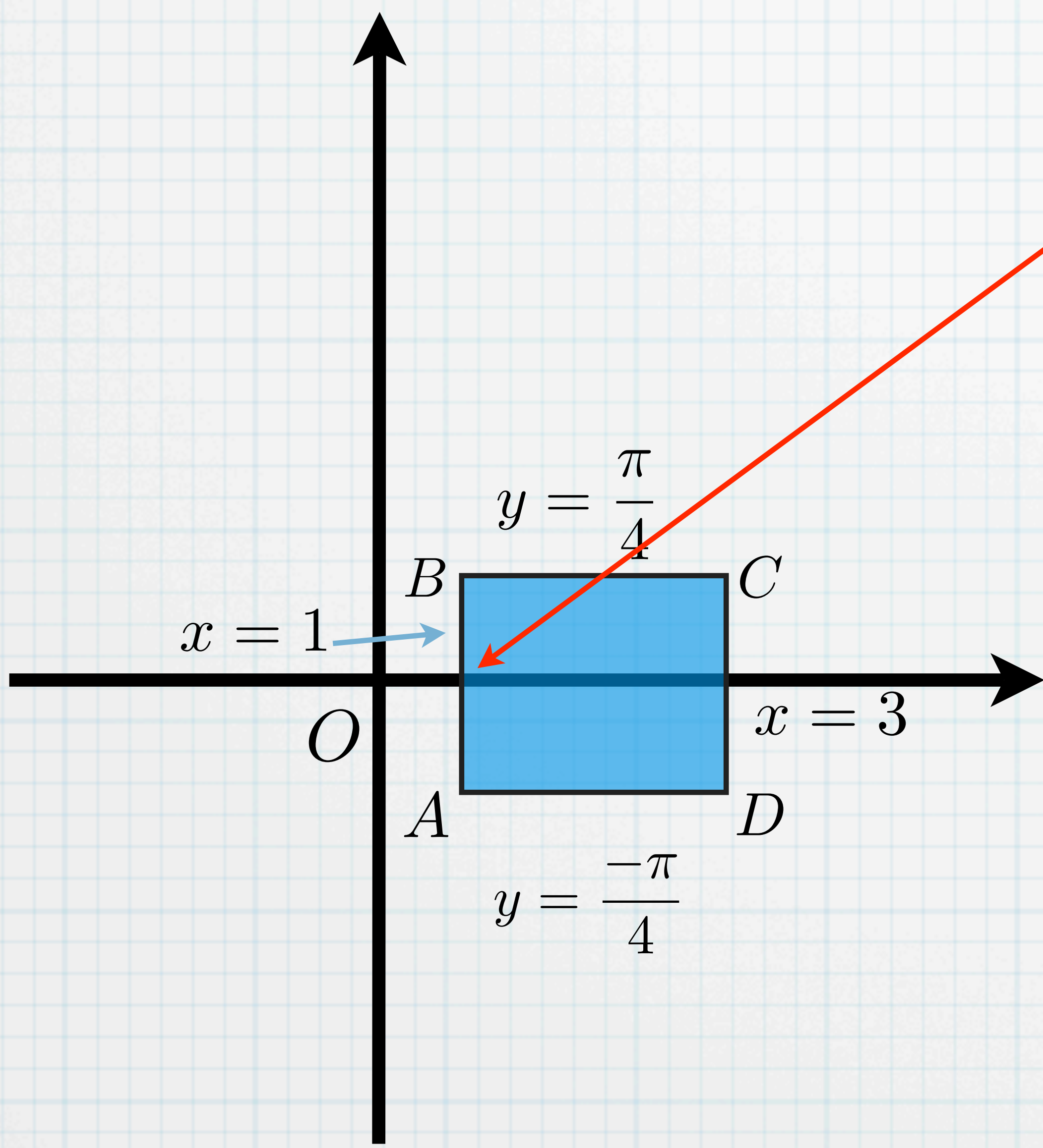
-5 es el mínimo absoluto en (1,2)




50
40
30
20
10
0
4
3
2
1
0
x
2,0
1,5
1,0
y
0,5
0,0

b) $f(x, y) = (4x - x^2)\cos(y)$

$$1 \leq x \leq 3, \ \frac{-\pi}{4} \leq y \leq \frac{\pi}{4}$$



i) En AB:

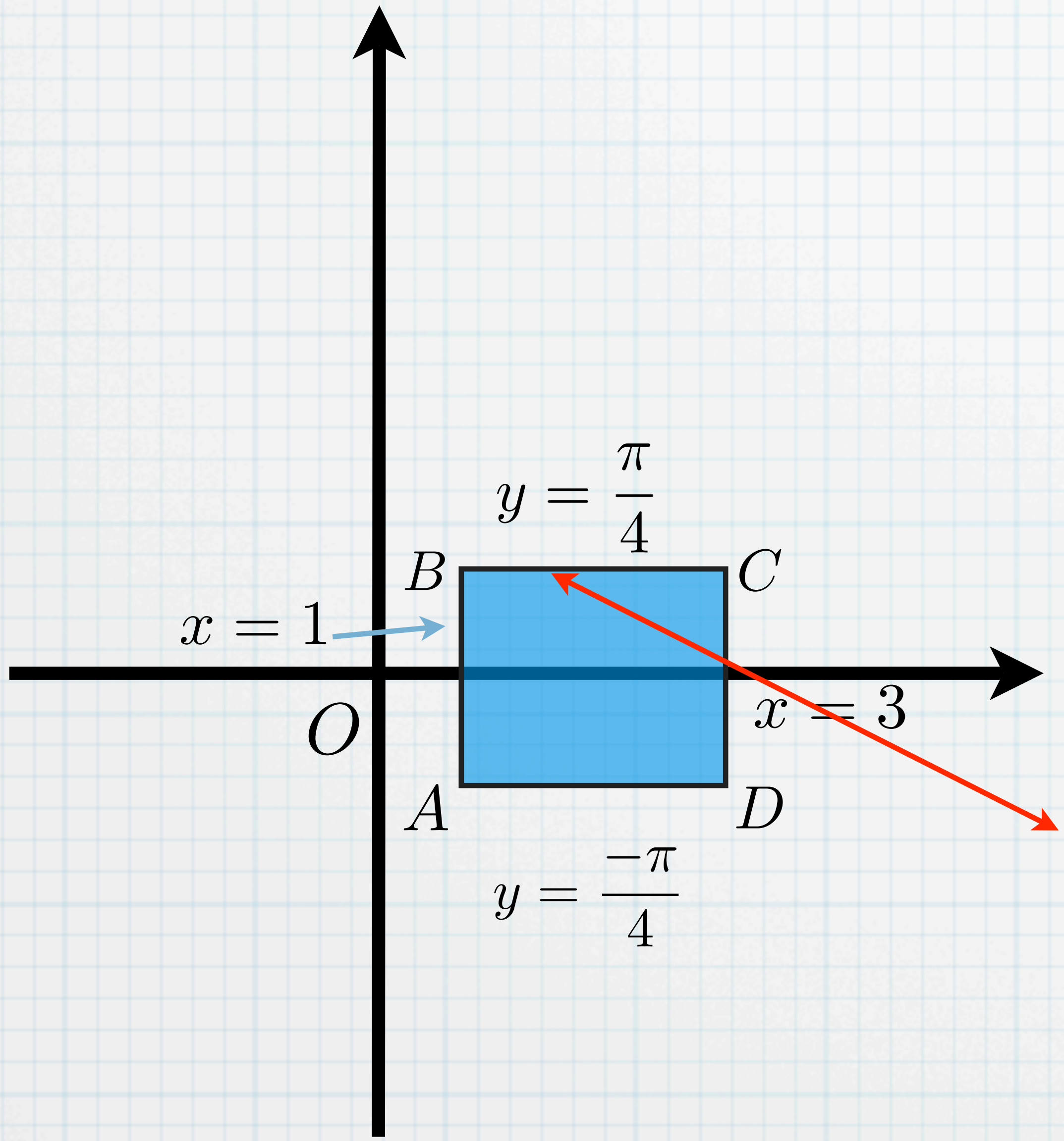
$$f(1, y) = 3\cos(y), \ \text{en} \ -\frac{\pi}{4} \leq y \leq \frac{\pi}{4}$$

$$f'(1, y) = -3\text{sen}(y) = 0 \Rightarrow y = 0, \ \text{y} \ x = 1$$

$$f(1, 0) = 3, \ f(1, \frac{-\pi}{4}) = \frac{3\sqrt{2}}{2}, \ f(1, \frac{\pi}{4}) = \frac{3\sqrt{2}}{2}$$

b) $f(x,y) = (4x - x^2)\cos(y)$

$$1 \le x \le 3, \ \frac{-\pi}{4} \le y \le \frac{\pi}{4}$$



i) En AB:

$$f(1,y) = 3\cos(y), \ \text{en} \ -\frac{\pi}{4} \le y \le \frac{\pi}{4}$$

$$f'(1,y) = -3\text{sen}(y) = 0 \Rightarrow y = 0, \ \text{y} \ x = 1$$

$$f(1,0) = 3, \ f(1, \frac{-\pi}{4}) = \frac{3\sqrt{2}}{2}, \ f(1, \frac{\pi}{4}) = \frac{3\sqrt{2}}{2}$$

ii) En BC:

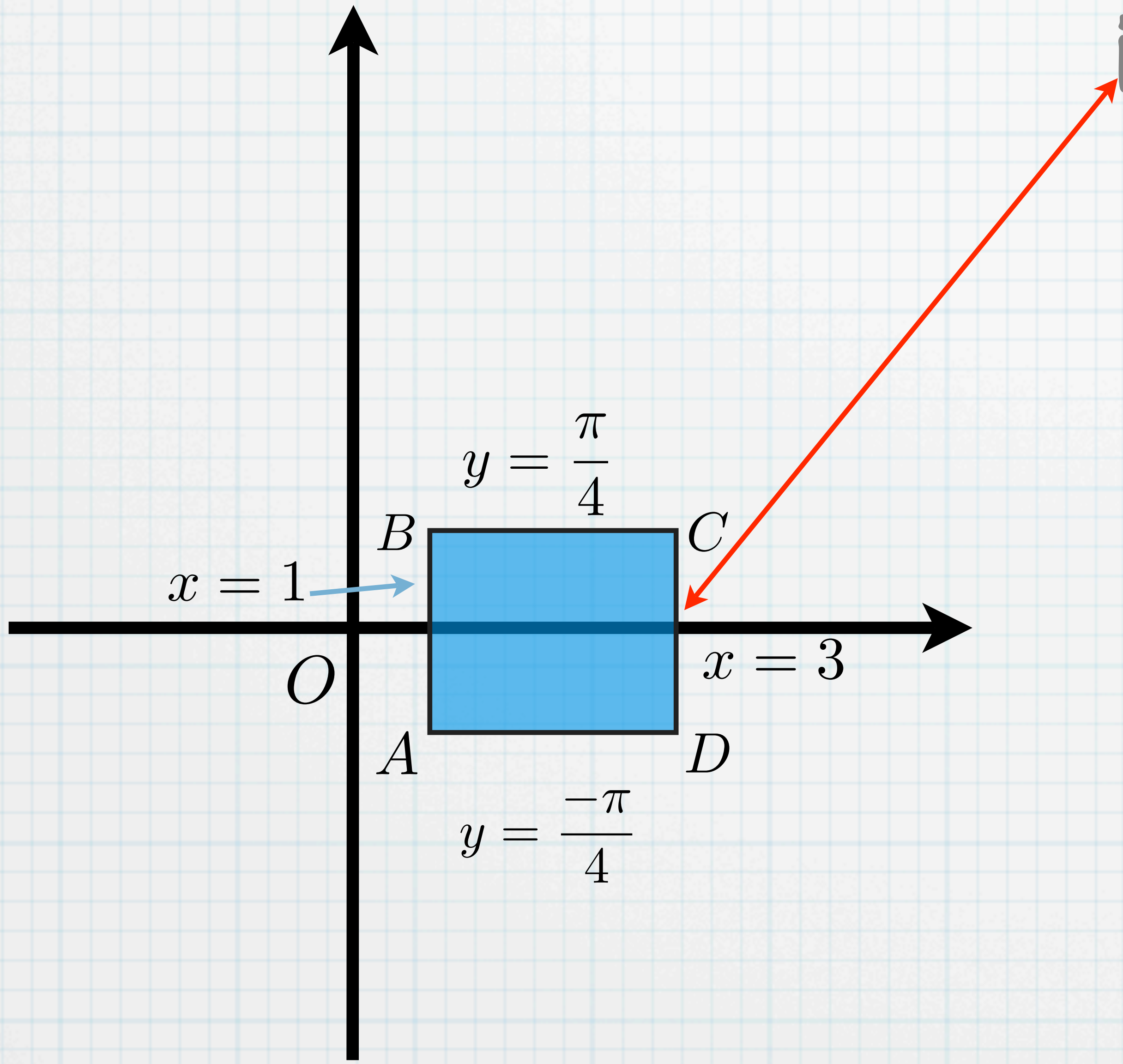
$$f(x, \frac{\pi}{4}) = \frac{\sqrt{2}}{2}\left(4x - x^2\right), \ \text{en} \ 1 \le x \le 3$$

$$f'(x, \frac{\pi}{4}) = \sqrt{2}\,(2 - x) = 0 \Rightarrow x = 2, \ \text{y} \ y = \frac{\pi}{4}$$

$$f(2, \frac{\pi}{4}) = 2\sqrt{2}, \ f(1, \frac{\pi}{4}) = \frac{3\sqrt{2}}{2}, \ f(3, \frac{\pi}{4}) = \frac{3\sqrt{2}}{2}$$

b) $f(x,y) = (4x - x^2)\cos(y)$

$$1 \leq x \leq 3, \ \frac{-\pi}{4} \leq y \leq \frac{\pi}{4}$$



iii) En CD:

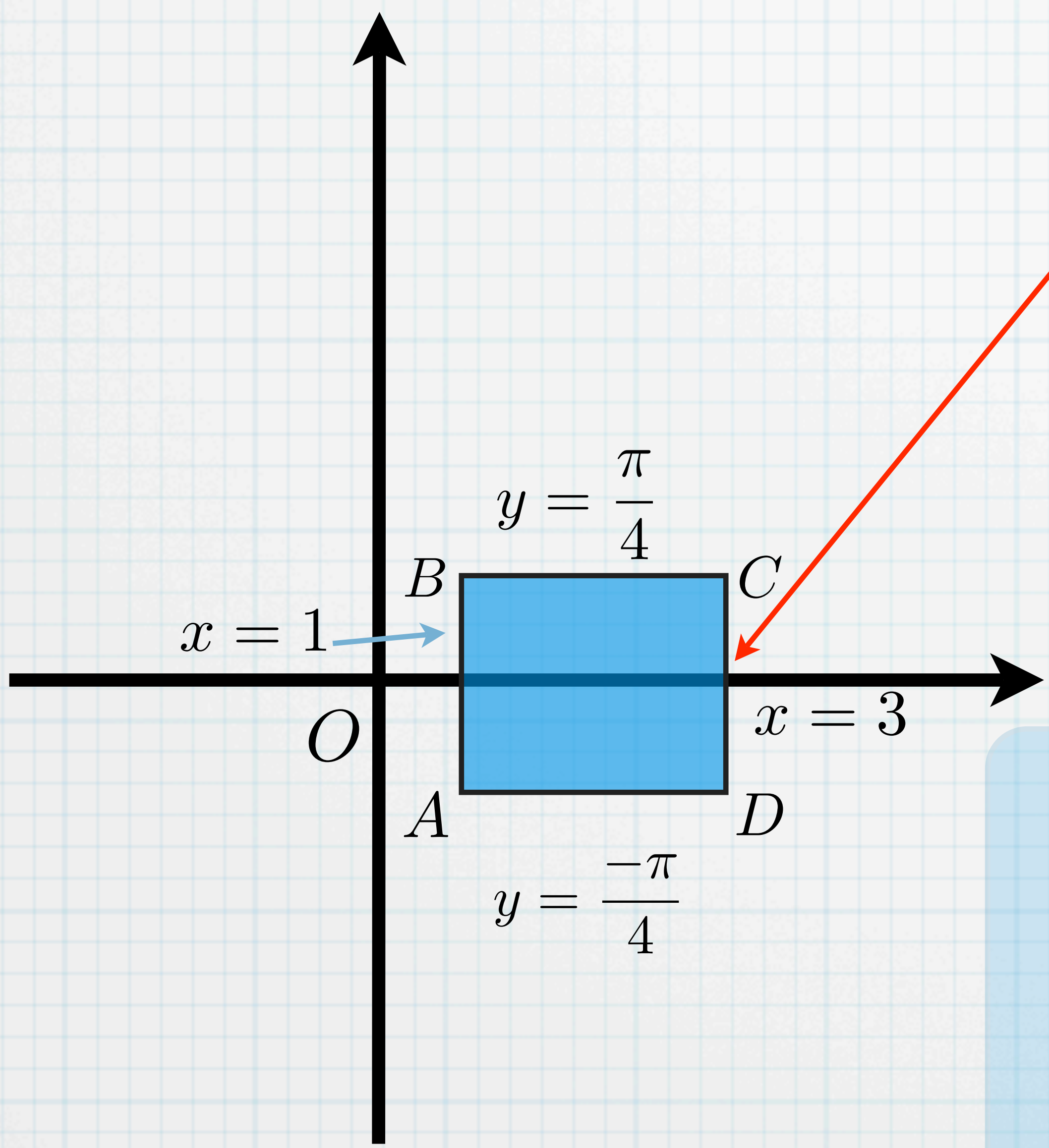
$$f(3,y) = 3\cos(y), \ \text{en} \ \frac{-\pi}{4} \leq y \leq \frac{\pi}{4}$$

$$f'(3,y) = -3\text{sen}(y) = 0 \Rightarrow y = 0, \ x = 3$$

$$f'(3,0) = 3, \ f(3, \frac{-\pi}{4}) = \frac{3\sqrt{2}}{2}, \ f(3, \frac{\pi}{4}) = \frac{3\sqrt{2}}{2}$$

b) $f(x, y) = (4x - x^2)\cos(y)$

$$1 \leq x \leq 3, \ \frac{-\pi}{4} \leq y \leq \frac{\pi}{4}$$



i) En CD:

$$f(3, y) = 3\cos(y), \ \text{en} \ \frac{-\pi}{4} \leq y \leq \frac{\pi}{4}$$

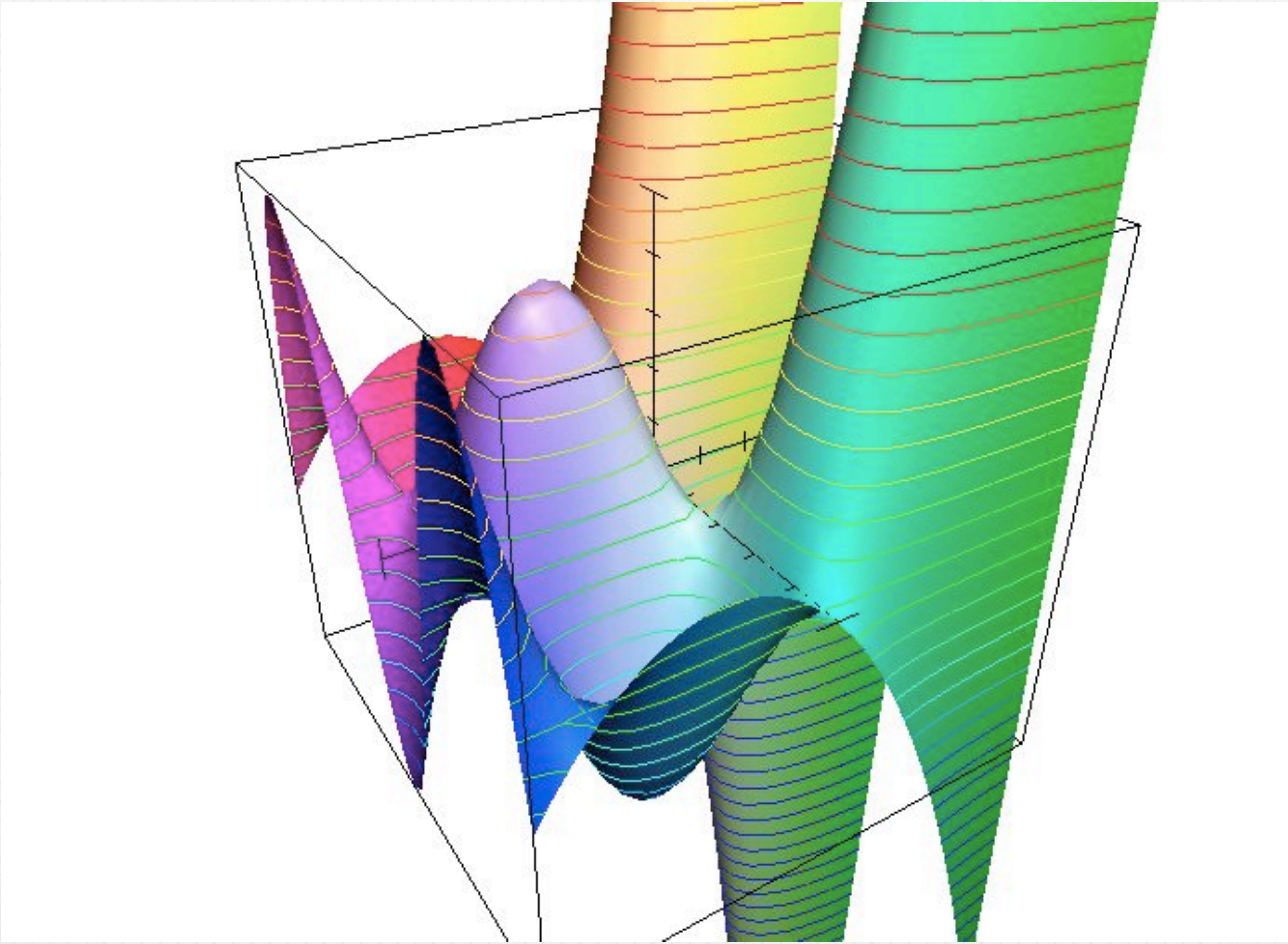
$$f'(3, y) = -3\text{sen}(y) = 0 \Rightarrow y = 0, \ x = 3$$

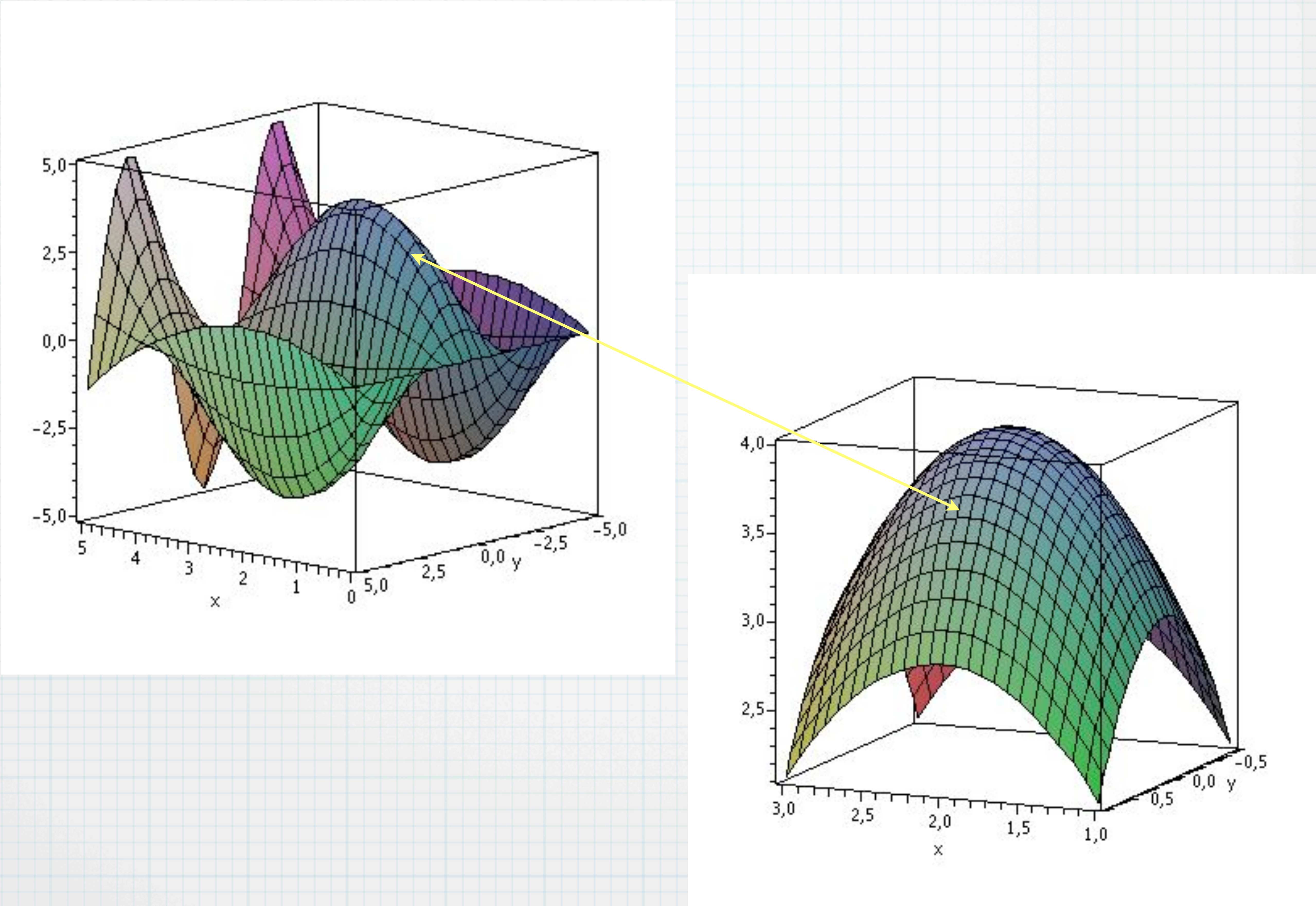
$$f'(3, 0) = 3, \ f(3, \frac{-\pi}{4}) = \frac{3\sqrt{2}}{2}, \ f(3, \frac{\pi}{4}) = \frac{3\sqrt{2}}{2}$$

iv) En DA:

v) En el interior:

Ejercicio.




5,0
2,5
0,0
-2,5
-5,0
5
4
3
2
1
0
x
5,0
2,5
0,0
y
-2,5
-5,0
4,0
3,5
3,0
2,5
3,0
2,5
2,0
1,5
1,0
x
0,5
0,0
y
-0,5

Ejercicio 2.

Encuentre las constantes a y b, satisfaciendo a<b, tal que:

$$\int_a^b \left(24 - 2x - x^2\right)\, dx$$

alcance su valor máximo

(r) a=-6, b=4